CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde A

Date:	17 April 2026
Time:	13.30 – 16.30
Questions:	6

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid (*see also question 1)*.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last two pages of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:						
Question	1	2	3	4	5	6
a	7	6	4	4	4	3
b	6	4	3	4	2	2
c	4	4	2	5	6	3
d			6		2	
Total	17	14	15	13	14	8

$\text{Grade} = \dfrac{\text{total points scored}}{9} + 1$

You will pass the exam if your grade is at least 5.5 .

Question 1 – Algebraic computations

Take a new answer sheet for every question!

*When you are asked to perform a computation **algebraically**, your computation should be fully worked out on paper. Reading function values from a table (including tables produced by a calculator) is not allowed in algebraic calculations. You can use a calculator for simple calculations and for approximations of numbers like* $\sqrt{2}$ *and* $\log(3)$.
Unless stated otherwise, all computations in this exam have to be performed algebraically.

The function f is given by $f(x) = x^4 - 10x^3 + 28x^2 - 64$.

7pt a Compute algebraically the minimum value of $f(x)$.

The function g is given by

$$g(x) = \frac{2 - x^2}{x + 2}$$

6pt b Compute algebraically the value(s) of x for which the graph of g in the point $A(x, g(x))$ is parallel to the line with equation $y = x - 4$.

4pt c Compute algebraically the sum of the powers of 3 that are between 10 and 10 million.
Hint: The largest power of 3 smaller than $10\ 000\ 000$ *is* $3^{14} = 4\ 782\ 969$, *so you have to compute* $27 + 81 + \cdots + 1\ 594\ 323 + 4\ 782\ 969$.

Question 2 – Football on your smartphone

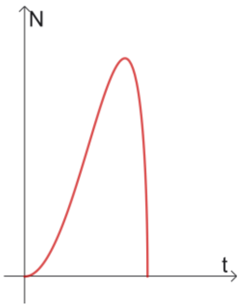
Take a new answer sheet for every question!

On 1 June, an app will be launched with information on the performance of the Dutch national football team at the world cup. There are two models for the number of users of this app in the months after the launch.

In the first model, the number of users is given by

$$N_1 = t^2 \cdot \sqrt{8 - 3t^2}$$

In this formula, N_1 is the number of users <u>in millions</u> according to the first model and *t* is the time in months with $t = 0$ on 1 June.
The figure below shows the graph that represents the relation between N_1 and t.



6pt a Compute algebraically the time at which, according to this first model, the number of users of this app is maximal.

In the second model, the number of users is given by

$$N_2 = \mathrm{e}^{-t^2+4t-2}$$

In this formula, N_2 is the number of users <u>in millions</u> according to the second model and *t* is again the time in months with $t = 0$ on 1 June.

4pt b Compute algebraically by what percentage the number of users of the app at $t = 1$ according to the second model differs from the number of users of the app at $t = 1$ according to the first model.

4pt c Use the derivative $\frac{\mathrm{d}N_2}{\mathrm{d}t}$ to determine whether, according to the second model, the number of users of this app will be increasing or decreasing on 16 July (that is at $t = 1.5$).

Question 3 – Delivering packages

Take a new answer sheet for every question!

An online store has been using delivery service S&S (Safe and Secure) for years to deliver ordered packages to customers. At S&S, the delivery time for the packages is normally distributed with an average of $\mu = 60$ hours and a standard deviation of $\sigma = 12$ hours.

4pt a Use the figure at the bottom of this page to compute the percentage of these packages with a delivery time between 60 and 78 hours.

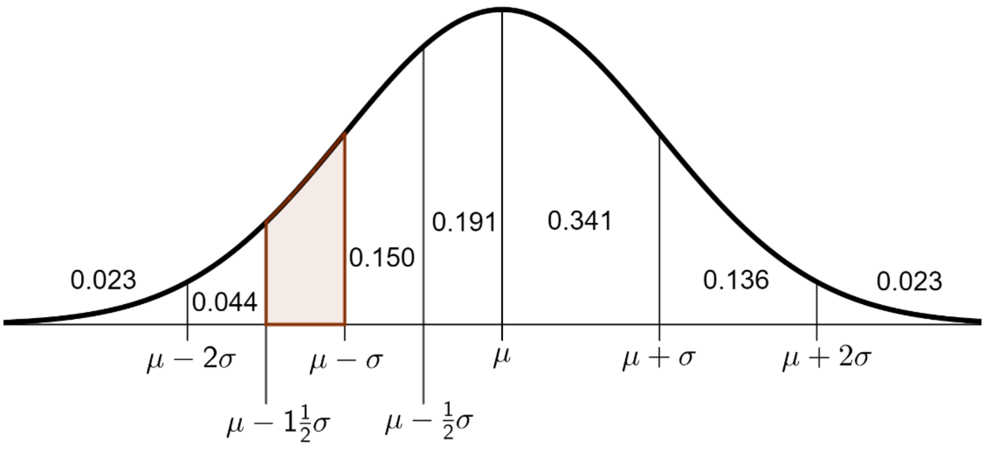
The online store is looking for a delivery service that better meets their needs.
With delivery service B&R (better and reliable), the delivery time for the packages is normally distributed with an average of $\mu = 62$ hours and a standard deviation of $\sigma = 8$ hours.

3pt b Which of these two delivery services has the highest percentage of packages with a delivery time of less than 75 hours?
Explain your answer with a computation or reasoning!

Delivery service G&C (good and cheap) claims that their average delivery time is 60 hours. Since G&C is much cheaper than S&S, the online store suspects that the delivery time at G&C is longer than 60 hours. To test this, 40 packages are sent independently of each other using G&C. In this testing procedure, we assume that the delivery times of the packages are normally distributed with a standard deviation of 12 hours and we assume a significance level of $\alpha = 0.05$.

2pt c State the null hypothesis and the alternative hypothesis for this test procedure.

6pt d What is the conclusion of this test procedure if the average delivery time of these 40 packages is 64 hours?



A normal probability distribution X
The area of the shaded region represents $P\left(\mu - 1\frac{1}{2}\sigma < X < \mu - \sigma\right) = 0.092$

Question 4 – An intermezzo at a bingo night

Take a new answer sheet for every question!

During a break on a bingo night, a short game can be played.
In this game, participants randomly pick four different coins from a bowl containing 15 red, 15 blue, 15 green and 5 gold coloured coins.

4pt a Compute the probability that the four coins that a participant picks, have four different colours.
Give your answer rounded to 4 digits behind the decimal point.

In this game, the participants win a prize when they pick at least one gold coloured coin. The probability that a participant wins a prize is approximately 0.3530.

4pt b Compute this probability rounded to 6 digits behind the decimal point.

One participant plays this game 10 times.

5pt c Compute the probability that this participant wins a prize in precisely 4 of these 10 times.
Give your answer rounded to 4 digits behind the decimal point.

Question 5 is on the next page

Question 5 – Two growth models

Take a new answer sheet for every question!

In the spring of 1995, cell phones were introduced in country C.
In the first ten years after the introduction, that is from 1995 to 2005, the percentage of the population of country C that used (one or more) cell phones grew exponentially.
On 1 July 1996, 0.96% of the population used cell phones.
On 1 July 2000, 4.86% of the population used cell phones.

4pt **a** Compute algebraically the percentage of the population of country C that used cell phones on 1 January 1999.

2pt **b** Explain why the exponential growth model cannot be used for the next period of ten years, that is from 2005 to 2015.

According to the cell phone provider Letrebil, the percentage of the population of country C that used cell phones in the years after 2005 is given by the formula

$$P = 9(10 - 5.9\mathrm{e}^{-0.28t})$$

(t in years with $t = 0$ on 1 July 2005)

6pt **c** Compute algebraically the time (year and month) at which, according to the formula of Letrebil, 81% of the population of country C used cell phones.

2pt **d** According to this formula, what percentage of the population of country C will use cell phones in the long run?

Question 6 is on the next page

Question 6 – Sunshine hours

Take a new answer sheet for every question!

A solar park has recorded the long-term average number of sunshine hours per day for every day of the year. This number can be approximated using the formula

$$S = 4.6 + 2.8\sin(0.017214(t - 75))$$

In this formula, t is the number of the day, for example $t = 1$ on 1 January and $t = 365$ on 31 December.

In this question, we neglect any leap day.

3pt a Explain the factor 0.017214 in this formula.

2pt b Determine the maximum and the minimum average number of sunshine hours per day according to this formula.

3pt c Compute algebraically the number of the day on which the average number of sunshine hours is maximal according to the formula given above.

End of the exam.

When you have finished the exam, check whether your ***name*** *and the* ***question number*** *are on every answer sheet.*

Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.

What should ***not*** *be in the folder:*
- empty sheets, please leave them on your table;
- sheets with only your name on it, please take them with you;
- scrap paper;
- these questions.

This is the only way we can ensure a smooth correction of your exam work.

Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde A

Quadratic equations

The solutions of the equation $ax^2 + bx + c = 0$ with $a \neq 0$ and $b^2 - 4ac \geq 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

Differentiation

Rule	**function**	**derivative function**
Sum rule	$s(x) = f(x) + g(x)$	$s'(x) = f'(x) + g'(x)$
Product rule	$p(x) = f(x) \cdot g(x)$	$p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$q(x) = \dfrac{f(x)}{g(x)}$	$q'(x) = \dfrac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$
Chain rule	$k(x) = f(g(x))$	$k'(x) = f'\left(g(x)\right) \cdot g'(x)$ or $\dfrac{dk}{dx} = \dfrac{df}{dg} \cdot \dfrac{dg}{dx}$

Logarithms

Rule	**conditions**
${}^{g}\log a + {}^{g}\log b = {}^{g}\log ab$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a - {}^{g}\log b = {}^{g}\log \dfrac{a}{b}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad b > 0$
${}^{g}\log a^p = p \cdot {}^{g}\log a$	$g > 0, \quad g \neq 1, \quad a > 0$
${}^{g}\log a = \dfrac{{}^{p}\log a}{{}^{p}\log g}$	$g > 0, \quad g \neq 1, \quad a > 0, \quad p > 0, \quad p \neq 1$

Arithmetic and geometric sequences

Arithmetic sequence:	$Sum = \frac{1}{2} \cdot number\ of\ terms \cdot (u_e + u_l)$
Geometric sequence:	$Sum = \dfrac{u_{l+1} - u_e}{r - 1} \quad (r \neq 1)$
In both formulas:	*e* = number first term of the sum; *l* = number last term of the sum

More formulas on the next page.

Formula list wiskunde A (continued)

Probability

If *X* and *Y* are any random variables, then: $E(X+Y) = E(X) + E(Y)$
If furthermore *X* and *Y* are independent, then: $\sigma(X+Y) = \sqrt{\sigma^2(X) + \sigma^2(Y)}$

$\sqrt{n}$-law:
For *n* independent repetitions of the same experiment where the result of each experiment is a random variable *X*, the sum of the results is a random variable *S* and the mean of the results is a random variable $\overline{X}$.

$E(S) = n \cdot E(X)$	$\sigma(S) = \sqrt{n} \cdot \sigma(X)$
$E(\overline{X}) = E(X)$	$\sigma(\overline{X}) = \frac{\sigma(X)}{\sqrt{n}}$

Binomial Distribution

If *X* has a binomial distribution with parameters *n* (number of experiments) and *p* (probability of success at each experiment), then

$P(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \quad \text{with} \quad k = 0, 1, 2, \ldots, n$

Expected value: $E(X) = np$ Standard deviation: $\sigma(X) = \sqrt{n \cdot p \cdot (1-p)}$

n and *p* are the parameters of the binomial distribution

Normal Distribution

If *X* is a normally distributed random variable with mean μ and standard deviation σ, then

$Z = \frac{X-\mu}{\sigma}$ has a standard normal distribution and $P(X < g) = P\left(Z < \frac{g-\mu}{\sigma}\right)$

μ and σ are the parameters of the normal distribution.

Hypothesis testing

In a testing procedure where the test statistic *T* is normally distributed with mean μ_T standard deviation σ_T the boundaries of the rejection region (the critical region) are:

α	left sided	right sided	two sided
0.05	$g = \mu_T - 1.645\sigma_T$	$g = \mu_T + 1.645\sigma_T$	$g_l = \mu_T - 1.96\sigma_T$
			$g_r = \mu_T + 1.96\sigma_T$
0.01	$g = \mu_T - 2.33\sigma_T$	$g = \mu_T + 2.33\sigma_T$	$g_l = \mu_T - 2.58\sigma_T$
			$g_r = \mu_T + 2.58\sigma_T$

